QMC and non-negative local discrepancy

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Quasi-Monte Carlo (QMC) rules are equal-weight quadrature rules for approximating integrals of functions depending on many variables. One of the main challenges when studying QMC rules is to find reliable error bounds. In this talk, we present a way of finding a non-asymptotic and computable upper bound for the integral of a function f over $[0,1]^d$. Indeed, let $f : [0,1]^d \to \mathbb{R}$ be a completely monotone integrand and let points $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{n-1} \in [0,1]^d$ have a non-negative local discrepancy (NNLD) everywhere in $[0,1]^d$. In such a situation, we can use the points $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{n-1}$ in a QMC rule and obtain the desired bound for the integral of f. An analogous non-positive local discrepancy (NPLD) property provides a computable lower bound.

We will also discuss which point sets are candidates for having the NNLD or NPLD property.

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